# SUCCESSIONI

**Teorema.** *Sia* $\{a_n\}$ *monotona non decrescente. Allora* $\{a_n\}$ *non oscilla e*

$$\lim_n a_n = \sup_n a_n \,;$$

*sia* $\{a_n\}$ *monotona non crescente. Allora* $\{a_n\}$ *non oscilla e*

$$\lim_n a_n = \inf_n a_n \,.$$

**Dim.** (caso non decrescente).

$1^{\text{mo}}$ *caso*: $\{a_n\}$ <u>non</u> limitata (superiormente), cioè $\sup_n a_n = +\infty$. Questo significa che:

$$\text{NON VALE } : \exists\, M > 0 : \ \forall\, n \in \mathbb{N} \quad a_n \leq M$$

$$\Updownarrow$$

$$\underline{\forall\, M > 0} \ \underline{\exists m \in \mathbb{N} :} \quad a_m > M$$

$\{a_n\}$ non decrescente $\Rightarrow$ $\underline{\forall\, n \geq m}$ $\underline{a_n \geq a_m > M}$

$\Rightarrow$ $a_n \to +\infty (= \sup_n a_n)$ .

$2^{\text{o}}$ *caso*: $\{a_n\}$ limitata (superiormente), cioè $\sup_n a_n = \Lambda \in \mathbb{R}$. Allora:

- $a_n \leq \Lambda$, $\forall\, n \in \mathbb{N}$;
- $\underline{\forall\, \varepsilon > 0}$ $\underline{\exists\, m_\varepsilon \in \mathbb{N}}$: $\Lambda - \varepsilon < a_{m_\varepsilon} \leq \Lambda$.

$\{a_n\}$ non decrescente

$\Rightarrow$ $\underline{\forall\, n \geq m_\varepsilon}$ $\underline{\Lambda - \varepsilon <} a_{m_\varepsilon} \leq \underline{a_n} \leq \Lambda \underline{< \Lambda + \varepsilon}$

$\Leftrightarrow$ $a_n \to \Lambda (= \sup_n a_n)$ .