FUNZIONI REALI

Operazioni sulle funzioni reali

Def. Siano f, g funzioni reali tali che $\operatorname{dom} f = \operatorname{dom} g = A$. Allora definiamo

$$\boxed{(f+g)(x) = f(x) + g(x),} \qquad \boxed{(f \cdot g)(x) = f(x) \cdot g(x)}$$

Le *funzioni costanti* 0 e 1 sono gli elementi neutri.

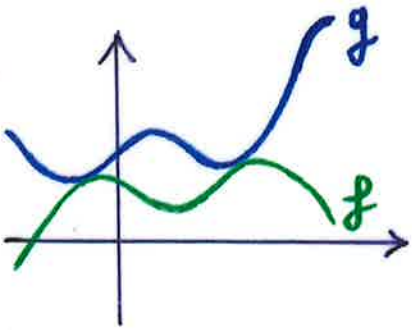
Opposto di f: $\quad (-f)(x) = -(f(x))$.

Se $g(x) \neq 0 \; \forall x \in A$ il *reciproco* di g *rispetto al prodotto*: $\left(\frac{1}{g}\right)(x) = \frac{1}{g(x)}$.

Nota: $\exists x \in A : \; g(x) = 0 \Longrightarrow \nexists \frac{1}{g}$

IMPORTANTE: non confondere il reciproco rispetto al prodotto con l'inverso rispetto alla *composizione*.

Ordine sulle funzioni reali

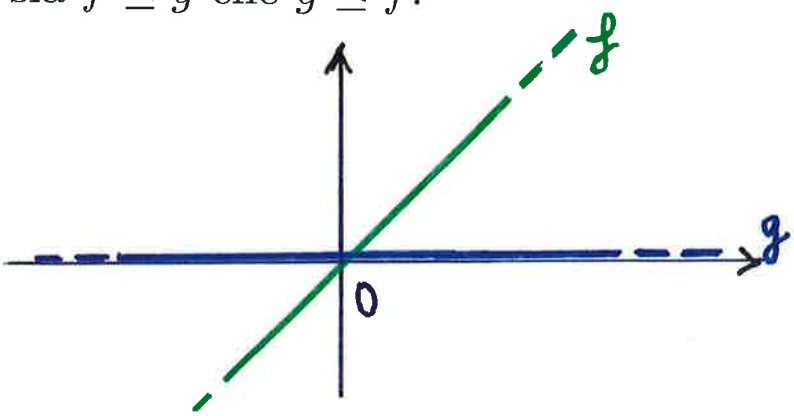


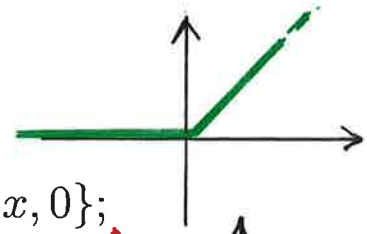
Def. $f \leq g \Longleftrightarrow f(x) \leq g(x) \; \forall x \in A$;

$f < g \Longleftrightarrow f(x) < g(x) \; \forall x \in A$.

Nota: l'ordine tra funzioni non è totale.

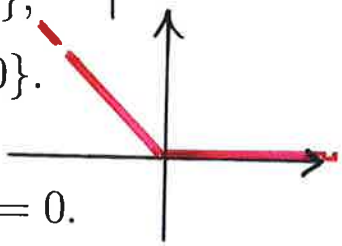
PER ESEMPIO: $f(x) = x$, $g(x) = 0$, $A = \mathbf{R}$. Sono false sia $f \leq g$ che $g \leq f$.





Def. *parte positiva* di $x \in \mathbf{R}$: $\quad (x)^+ = \max\{x, 0\}$;

parte negativa di $x \in \mathbf{R}$: $\quad (x)^- = \max\{-x, 0\}$.



ESEMPI: $(\pi)^+ = \pi$, $(3)^- = 0$, $(-e)^- = e$, $(-2)^+ = 0$.

ESERCIZIO: verificare che: $\quad (x)^+ \geq 0$, $(x)^- \geq 0$, $(x)^+ \cdot (x)^- = 0$, $(x)^+ - (x)^- = x$, $(x)^+ + (x)^- = |x|$.

Def. Siano $f, g : A \to \mathbf{R}$ funzioni reali. Allora definiamo:

$$f^+(x) = \Big(f(x)\Big)^+, \qquad f^-(x) = \Big(f(x)\Big)^-,$$

$$|f|(x) = |f(x)| = f^+(x) + f^-(x),$$

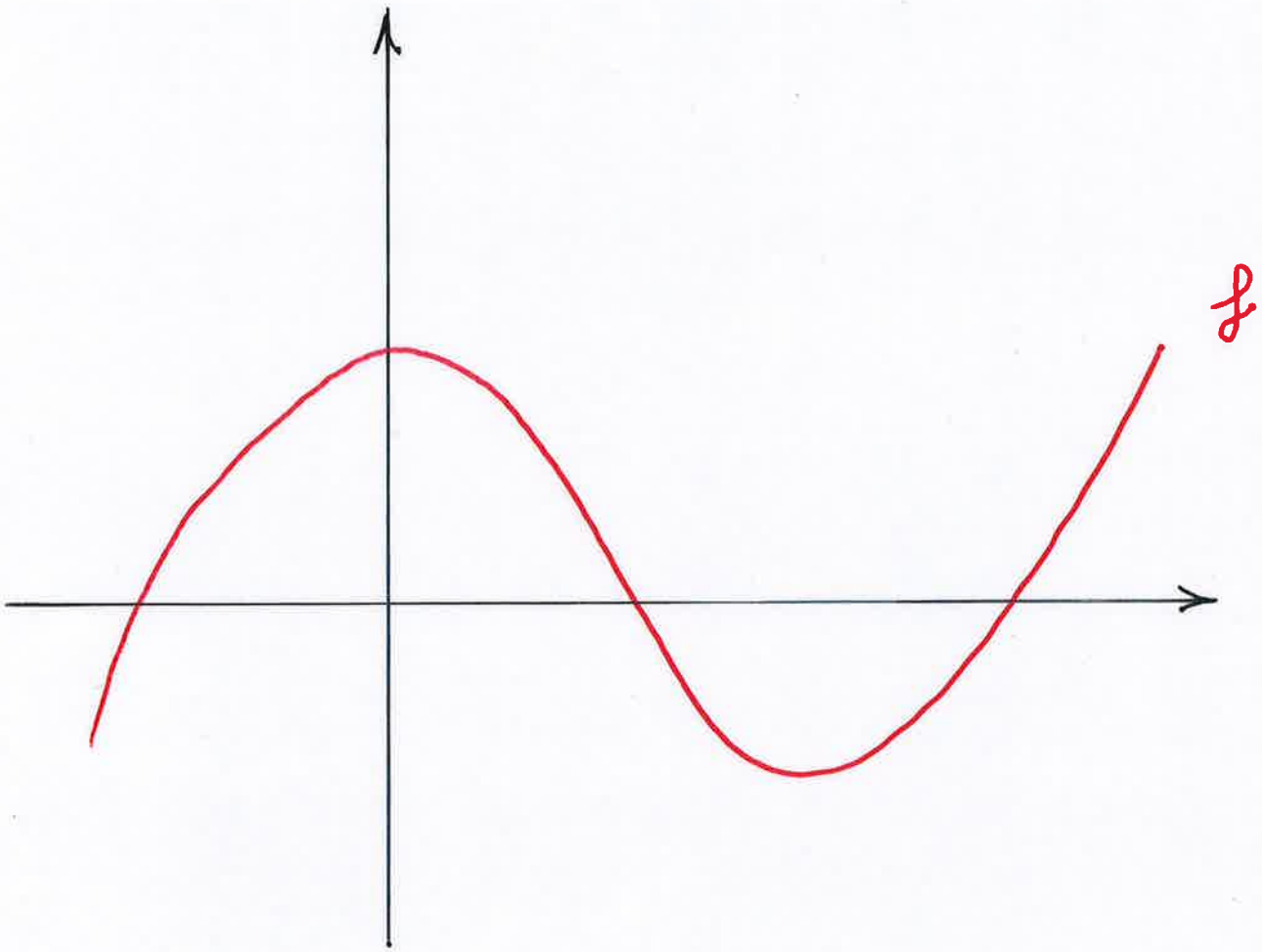
(osserviamo che $f(x) = f^+(x) - f^-(x)$)

$$\max\{f, g\}(x) = \max\{f(x), g(x)\},$$

$$\min\{f, g\}(x) = \min\{f(x), g(x)\}.$$

ESEMPIO: $f(x) = x$, $g(x) = -x$. Allora

$$\max\{f, g\}(x) = |x|, \qquad \min\{f, g\}(x) = -|x|.$$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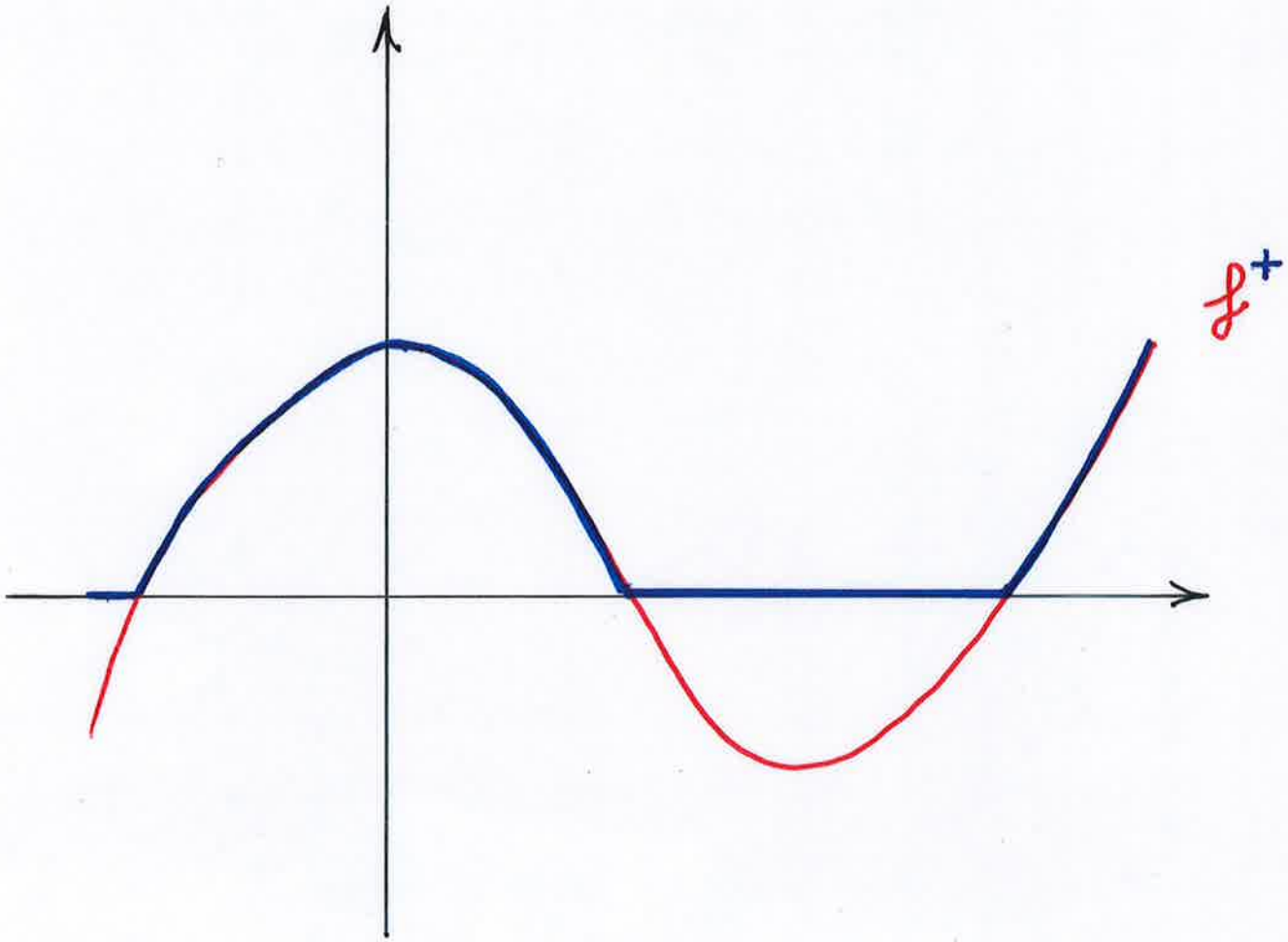
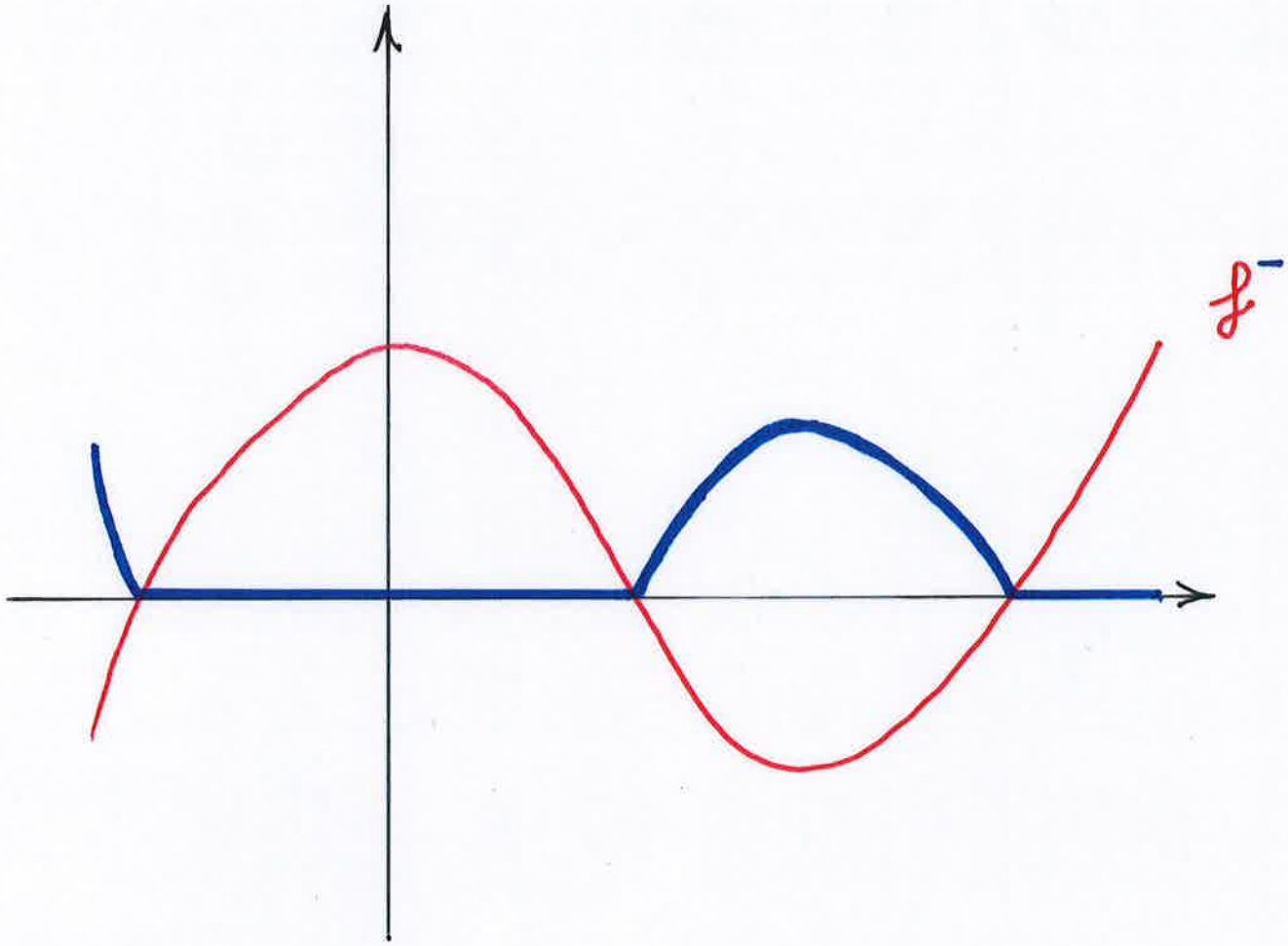
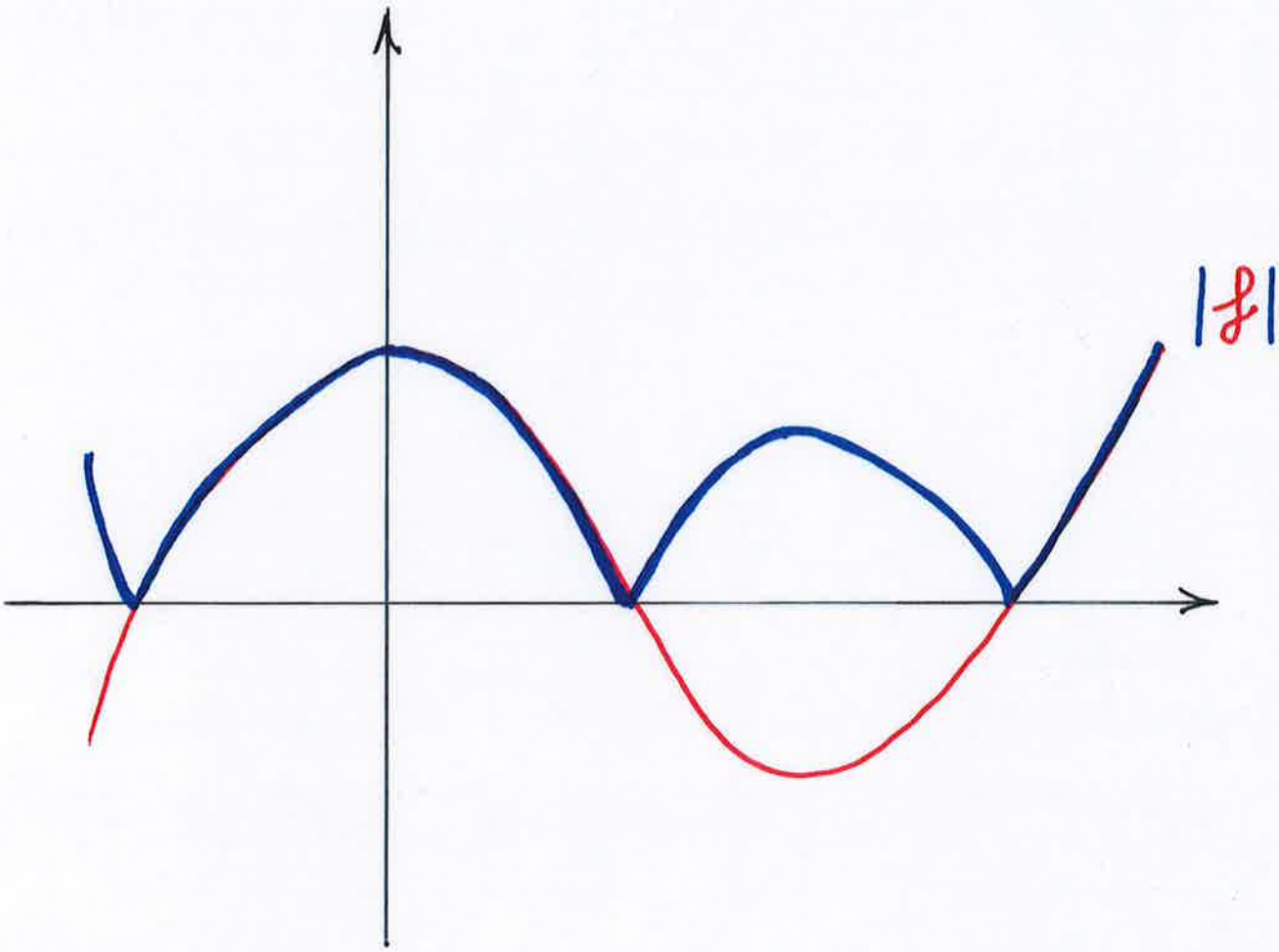


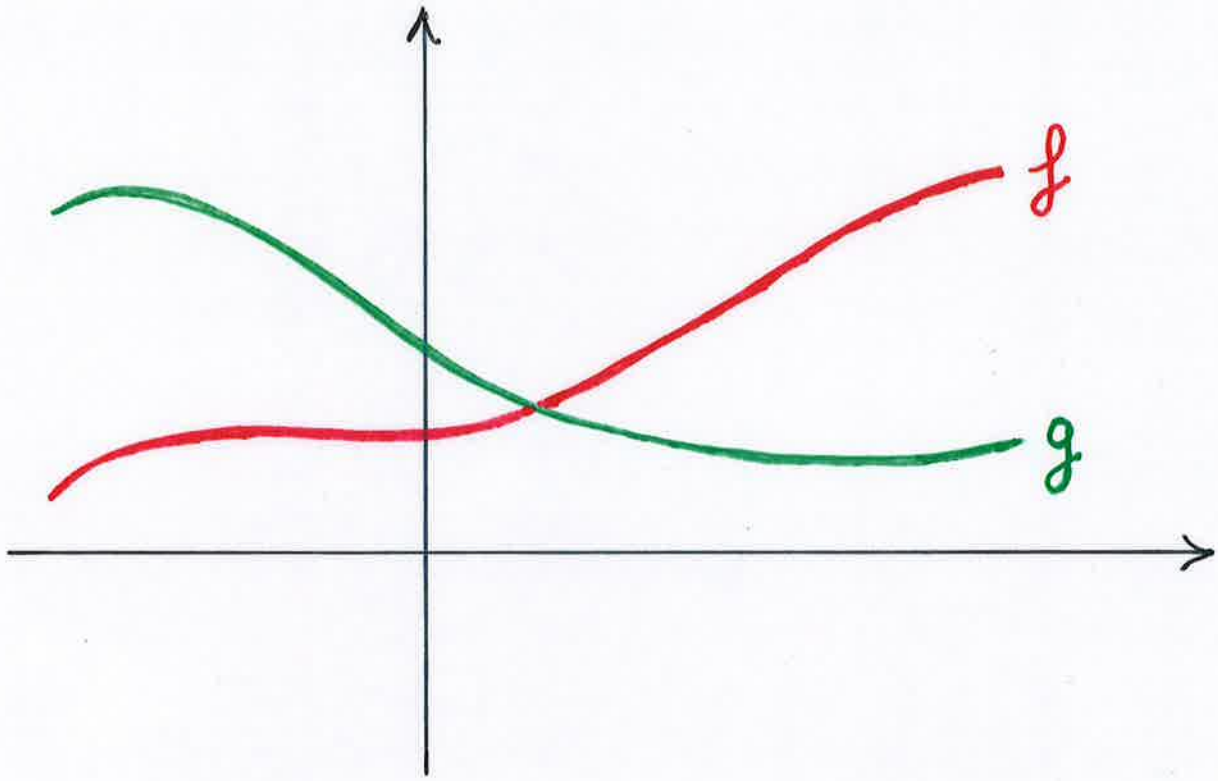
$$f^+(x) = (f(x))^+ = \max\{f(x), 0\}$$



$$f^-(x) = (f(x))^- = \max\{-f(x), 0\}$$



$|f|(x) = |f(x)|$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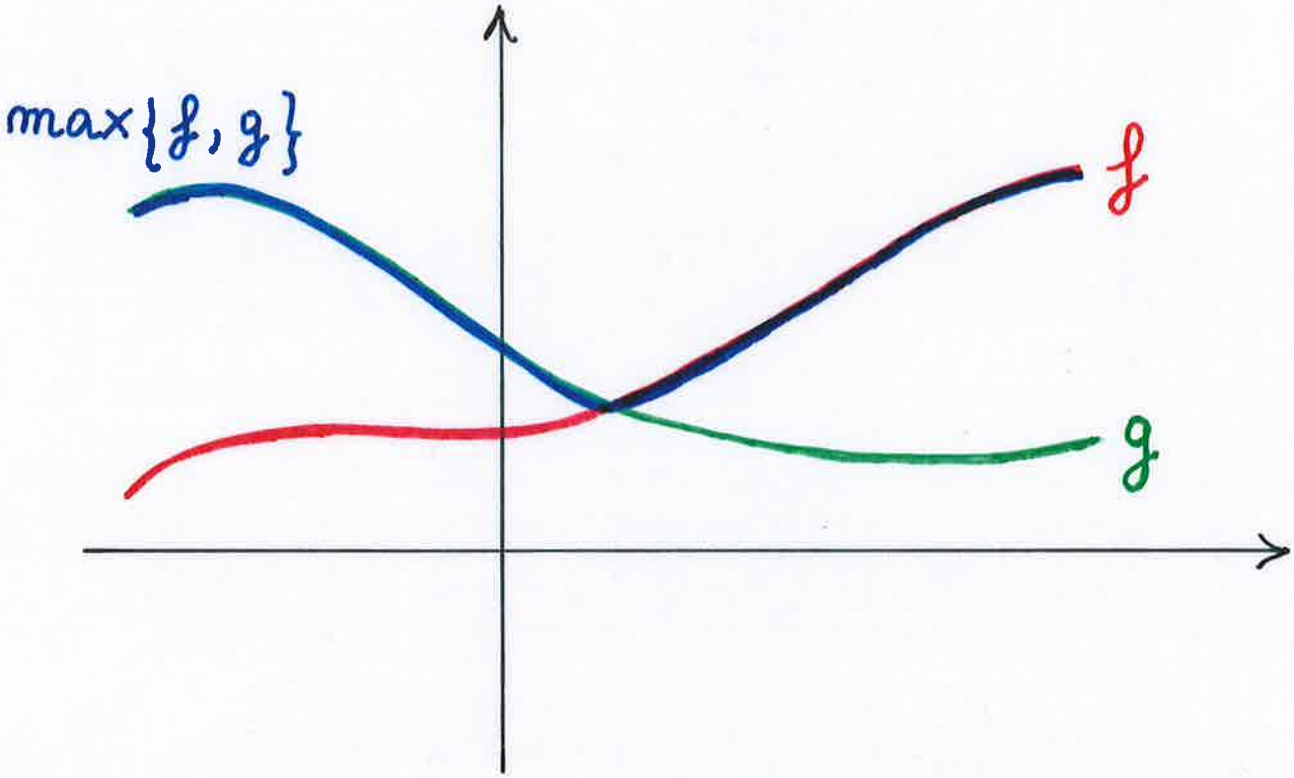
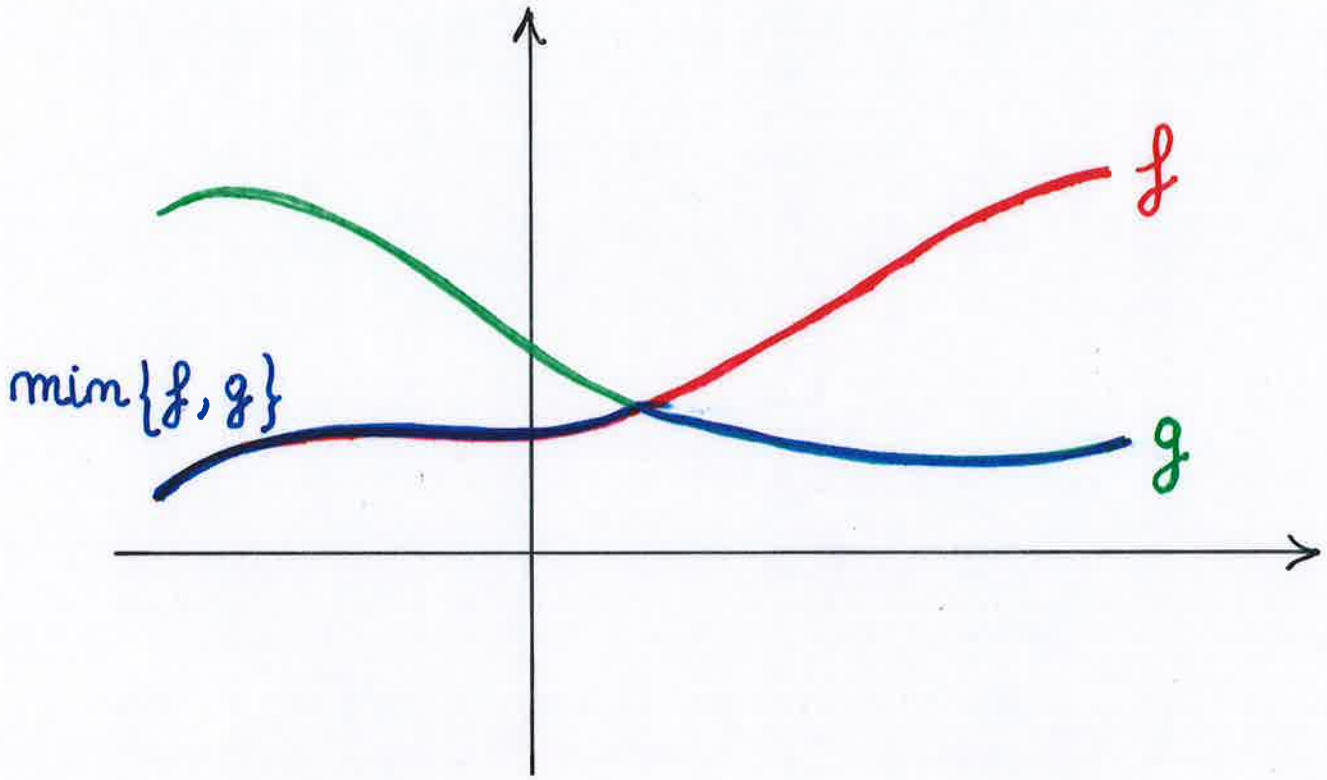


$$\max\{f, g\}(x) = \max\{f(x), g(x)\}$$



$$\min\{f, g\}(x) = \min\{f(x), g(x)\}$$